AP Calculus BC
Chapter 4 – AP Exam Problems

Extreme Values in an Interval

1. The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2, 4]$ occurs at $x=$

   A) 4 B) 2 C) 1 D) 0 E) -2

2. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

   A) 9 B) 12 C) 14 D) 21 E) 40

3. Let *f* be a function defined and continuous on the closed interval $[a, b]$. If *f* has a relative maximum at c and $a < c < b$, then which of the following statements is true?

   I. $f'(c)$ exists. II. If $f'(c)$ exists, then $f'(c) = 0$. III. If $f''(c)$ exists, then $f''(c) \le 0$.

   A) II only
   B) III only
   C) I and II only
   D) I and III only
   E) II and III only

4. (1993 AB4) Let *f* be the function defined by $f(x)=\ln(2+\sin x)$ for $\pi \le x \le 2\pi$. Find the absolute maximum value and the absolute minimum value of *f*. Show the analysis that leads to your conclusion.

Derivatives and Graphing

5. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

   A) One B) Two C) Three D) Five E) Nine

6. The first derivative of the function *f* is given by $f'(x)=\dfrac{\cos^2 x}{x}-\dfrac{1}{5}$. How many critical values does *f* have on the open interval (0, 10)? (Caculator)

   A) One B) Three C) Four D) Five E) Seven

7. The function *f* given by $f(x)=x^3+12x-24$ is

   A) increasing for $x<-2$, decreasing for $-2<x<2$, increasing for $x>2$
   B) decreasing for $x<0$, increasing for $x>0$
   C) increasing for all x
   D) decreasing for all x
   E) decreasing for $x<-2$, increasing for $-2<x<2$, decreasing for $x>2$

8. The function *f* is given by $f(x)=x^4+x^2-2$. On which of the following intervals is *f* increasing?

A) $\left(-\frac{1}{\sqrt{2}},\infty\right)$ C) $(0,\infty)$ E) $\left(-\infty,-\frac{1}{\sqrt{2}}\right)$
B) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ D) $(-\infty,0)$

9. If $f(x)=x^2e^x$, then the graph of *f* is decreasing for all *x* such that

A) $x<-2$ C) $x>-2$ E) $x>0$
B) $-2<x<0$ D) $x<0$

10. What are all values of *x* for which the function *f* defined by $f(x)=x^3+3x^2-9x+7$ is increasing?

A) $-3<x<1$ C) $x<-3$ or $x>1$ E) All real numbers
B) $-1<x<1$ D) $x<-1$ or $x>3$

11. The function defined by $f(x)=x^3-3x^2$ for all real numbers *x* has a relative maximum at *x* =

A) -2 B) 0 C) 1 D) 2 E) 4

12. At what values of *x* does $f(x)=3x^5-5x^3+15$ have a relative maximum?

A) -1 only C) 1 only E) -1, 0, and 1
B) 0 only D) -1 and 1 only

13. The <u>derivative</u> of *f* is $x^4(x-2)(x+3)$. At how many points will the graph of *f* have a relative maximum?

A) None C) Two E) Four
B) One D) Three

14. What is the minimum value of $f(x)=x\ln x$?

A) $-e$ B) -1 C) $-\frac{1}{e}$ D) 0 E) There is no minimum.

15. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true? (Calculator)

A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
C) f has a relative minima at $x = -2$ and at $x = 2$.
D) f has a relative maxima at $x = -2$ and at $x = 2$.
E) It cannot be determined if f has any relative extrema.

16. A polynomial $p(x)$ has a relative maximum at $(-2, 4)$, a relative minimum at $(1, 1)$, and a relative maximum at $(5, 7)$, and no other critical points. How many real zeros does $p(x)$ have?

A) One
B) Two
C) Three
D) Four
E) Five

17. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

A) $x < 0$ B) $x < 2$ C) $x < 5$ D) $x > 0$ E) $x > 2$

18. At what value of x does the graph of $y = \dfrac{1}{x^2} - \dfrac{1}{x^3}$ have a point of inflection?

A) 0 B) 1 C) 2 D) 3 E) At no value of x

19. What is the x-coordinate of the point of inflection on the graph of $y = \dfrac{1}{3}x^3 + 5x^2 + 24$?

A) 5 B) 0 C) $-\dfrac{10}{3}$ D) -5 E) -10

20. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when x =

A) -1 only
B) 2 only
C) -1 and 0 only
D) -1 and 2 only
E) -1, 0, and 2 only

21. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b?

A) -3 B) 0 C) 1 D) 3 E) Cannot be determined

22. If f is the function defined by $f(x)=3x^5-5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

A) -1 B) 0 C) 1 D) 0 and 1 E) -1, 0, and 1

23. If $f(x)=1+x^{2/3}$, which of the following is NOT true?

A) f is continuous for all real numbers.
B) f has a minimum at $x = 0$.
C) f is increasing for $x > 0$.
D) $f'(x)$ exists for all x.
E) $f''(x)$ is negative for $x > 0$.

24. Let f be a polynomial function with degree greater than 2. If $a\neq b$ and $f(a)=f(b)=1$, which of the following must be true for at least one value of x between a and b?

I. $f(x)=0$
II. $f'(x)=0$
III. $f''(x)=0$

A) None
B) I only
C) II only
D) I and II only
E) I, II, and III

25. (1988 AB4) Let f be the function defined by $f(x)=2xe^{-x}$ for all real numbers x.

a) Write an equation of the horizontal asymptote for the graph of f.
b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.
c) For what values of x is the graph of f concave down?
d) Using the results found in parts (a), (b), and (c), sketch the graph of $y=f(x)$.

26. (1993 AB1) Let f be the function given by $f(x)=x^3-5x^2+3x+k$, where k is a constant.

a) On what intervals is f increasing?
b) On what intervals is the graph of f concave downward?
c) Find the value of k for which f has 11 as its relative minimum.

27. (1993 AB4) Let f be the function defined by $f(x)=\ln(2+\sin x)$ for $\pi\leq x\leq 2\pi$.

a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to this conclusion.
b) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.

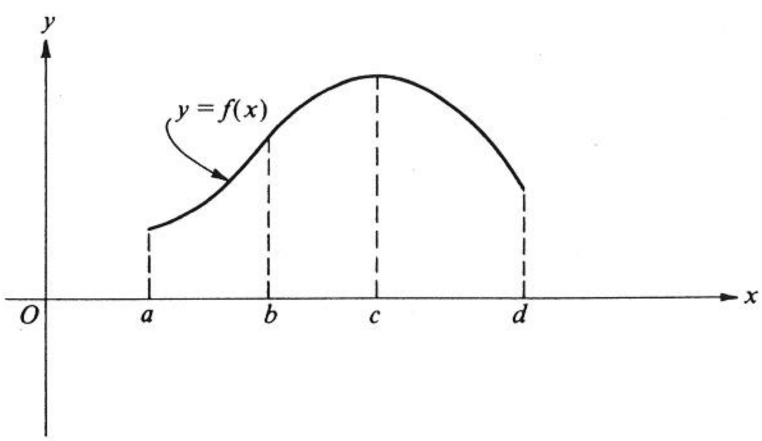
28. (1998 AB2) Let *f* be the function given by $f(x) = 2xe^{2x}$.

   a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   b) Find the absolute minimum value of *f*. Justify that your answer is an absolute minimum.
   c) What is the range of *f*?
   d) Consider the family of functions defined by $y = bxe^{bx}$, where *b* is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of *b*.

29. (1988 BC1) Let *f* be the function defined by $f(x) = \left(x^2 - 3\right) e^x$ for all real numbers *x*.

   a) For what values of *x* is *f* increasing?
   b) Find the *x*-coordinate of each point of inflection of *f*.
   c) Find the *x*-coordinates and *y*-coordinates of the point, if any, where $f(x)$ attains an absolute minimum.

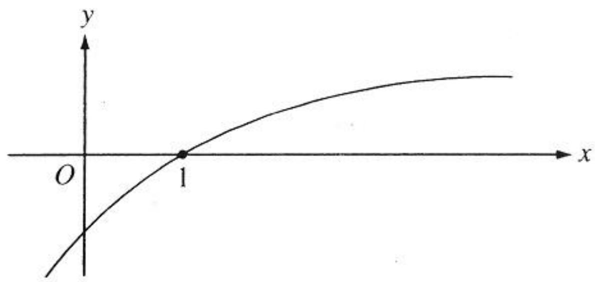
Interpreting Function and/or Derivative Graphs



30. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

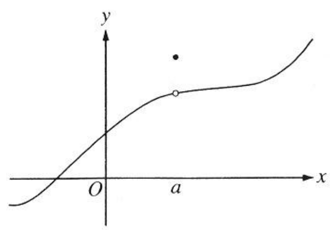
   I. $a < x < b$
   II. $b < x < c$
   III. $c < x < d$

A) I only
B) II only
C) III only
D) I and II
E) II and III



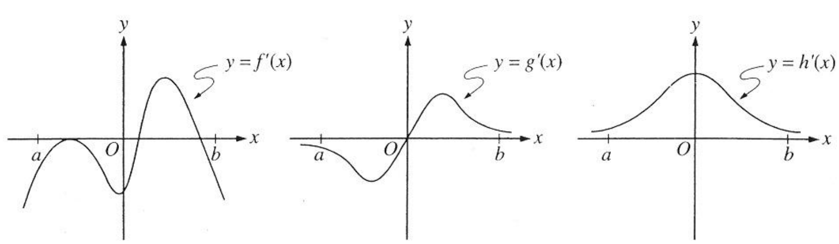
31. The graph of the twice-differentiable function *f* is shown in the figure above. Which of the following is true?

A) $f(1) < f'(1) < f''(1)$
B) $f(1) < f''(1) < f'(1)$
C) $f'(1) < f(1) < f''(1)$
D) $f''(1) < f(1) < f'(1)$
E) $f''(1) < f'(1) < f(1)$



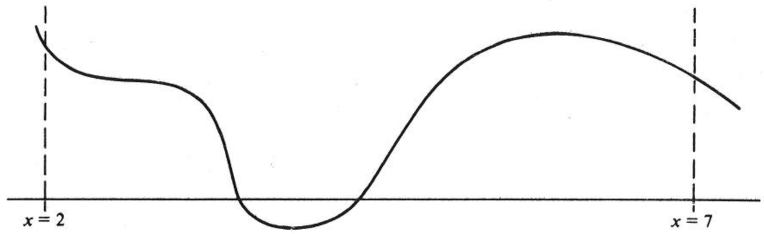
32. The graph of a function *f* is shown above. Which of the following statements about *f* is false?

A) *f* is continuous at $x = a$.
B) *f* has a relative maximum at at $x = a$.
C) $x = a$ is in the domain of *f*.
D) $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$.
E) $\lim_{x \to a} f(x)$ exists.



33. The graphs of the derivatives of the functions *f, g,* and *h* are shown above. Which of the functions *f, g,* or *h* have a relative maximum on the open interval $a<x<b$?

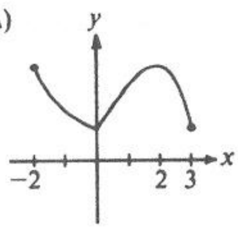
A) *f* only
B) *g* only
C) *h* only
D) *f* and *g* only
E) *f, g,* and *h*

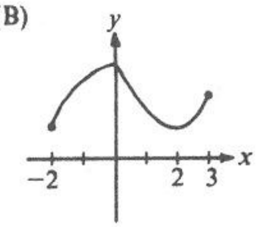


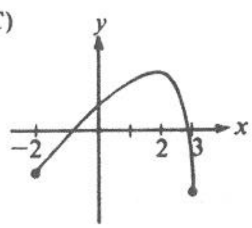
34. The graph of $y = f(x)$ on the closed interval [2, 7] is shown above. How many points of inflection does this graph have on this interval?

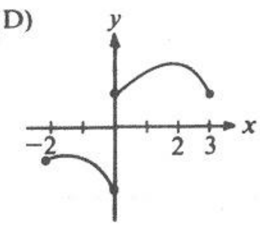
A) One B) Two C) Three D) Four E) Five

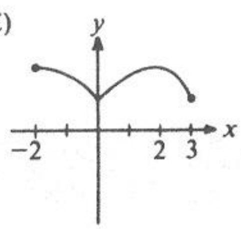
35. Let f be the function that is continuous on the closed interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$ and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?

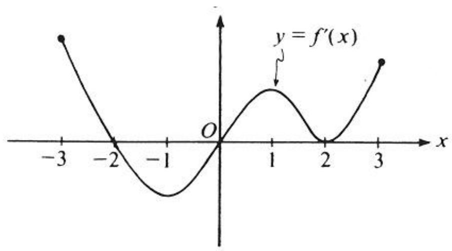
(A)


(B)


(C)


(D)


(E)




36. (1985 AB6) The figure above shows the graph of f', the <u>derivative</u> of a function *f*. The domain of the function *f* is the set if all *x* such that $-3 \le x \le 3$.

   a) For what values of *x*, $-3 < x < 3$, does *f* have a relative maximum? A relative minimum? Justify your answer.
   b) For what values of *x* is the graph of *f* concave up? Justify your answer.
   c) Use the information found in parts (a) and (b) and the fact that $f(-3) = 0$ to sketch a possible graph of *f*.

Mean Value Theorem for Derivatives

37. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number *c* in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be *c*?

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{6}$ D) π E) $\frac{3\pi}{2}$

38. Let *f* be the function given by $f(x) = x^3 - 3x^2$. What are all values of *c* that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval [0,3]?

A) 0 only B) 2 only C) 3 only D) 0 and 3 E) 2 and 3

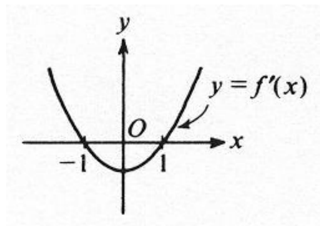
39. If *c* is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$, then *c* =

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{4}{3}$ E) 2

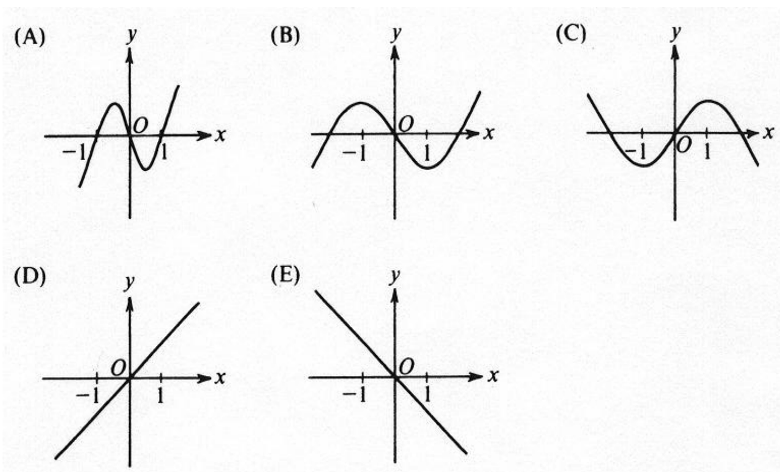
40. (1989 AB1) Let *f* be the function given by $f(x) = x^3 - 7x + 6$. Find the number *c* that satisfies the conclusion of the Mean Value Theorem for *f* on the closed interval [1, 3].

AP Calculus BC
Chapter 4 – AP Exam Problems

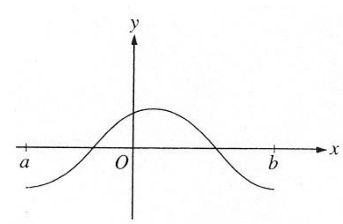
Function Graphs vs. Derivative Graphs

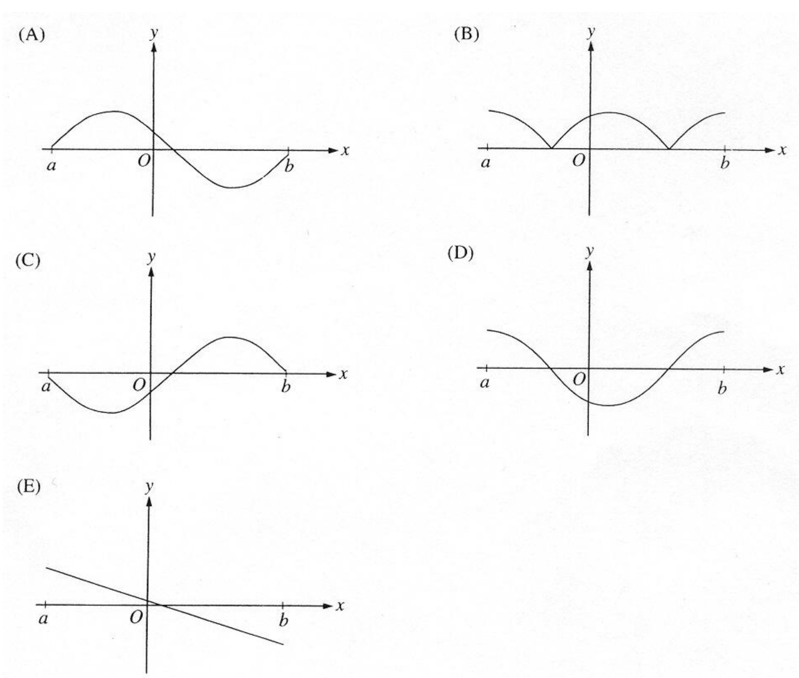


41. The graph of the <u>derivative</u> of *f* is shown in the figure above. Which of the following could be the graph of *f*?

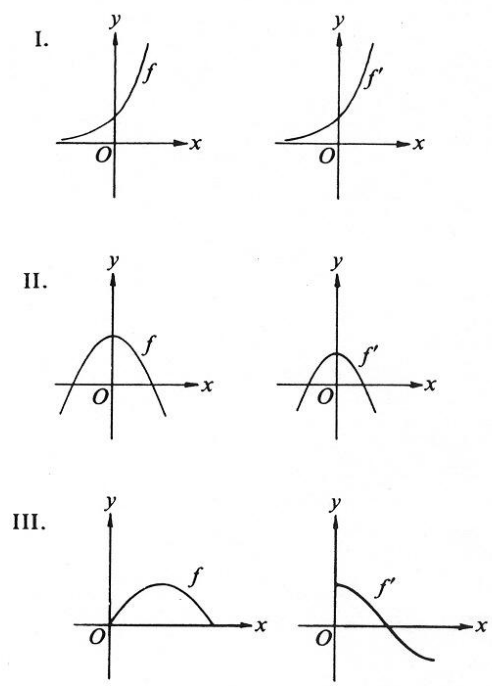


42. The graph of *f* is shown to the right. Which of the following could be the graph of the derivative of *f* ?



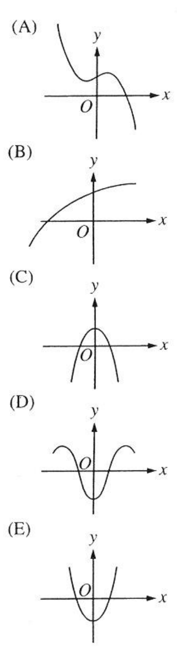


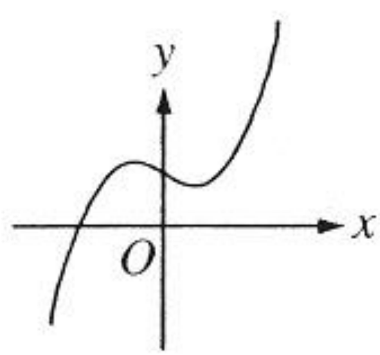
43. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?

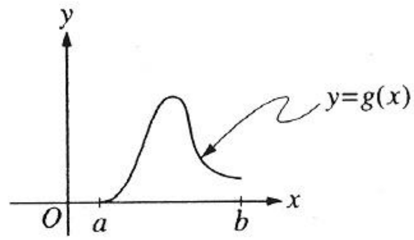


A) I only B) II only C) III only D) I and III E) II and III

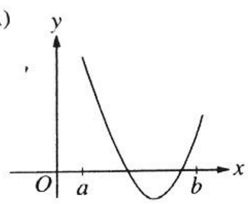
44. The graph of $y = h(x)$ is shown below to the right. Which of the following could be the graph of $y = h'(x)$?

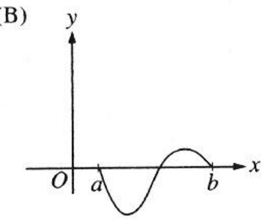


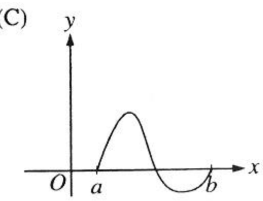


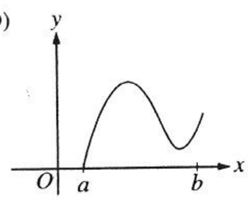


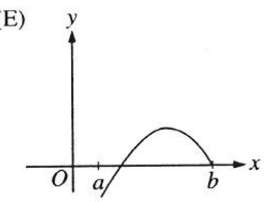
45. Let $g(x)=\int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


(B)


(C)


(D)


(E)


Optimization Problems

46. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

A) $2\sqrt[3]{2}$ B) $2\sqrt{2}$ C) $2\sqrt[3]{4}$ D) 4 E) 8

47. What is the area of the largest rectangle that can be inscribed in the ellipse $4x^2+9y^2=36$?

A) $6\sqrt{2}$ B) 12 C) 24 D) $24\sqrt{2}$ E) 36

48. Consider all right circular cylinders for which the sum of the height and the circumference is 30 centimeters. What is the radius of the one with maximum volume?

A) 3 cm B) 10 cm C) 20 cm D) $\frac{30}{\pi^2}$ cm E) $\frac{10}{\pi}$ cm

AP Calculus BC
Chapter 4 – AP Exam Problems

Linearization for Approximations

49. Let f be the function given by $f(x)=x^2-2x+3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

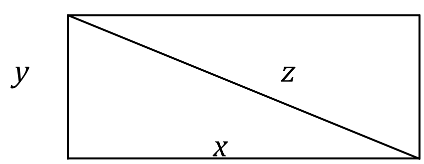
A) 2.4 B) 2.5 C) 2.6 D) 2.7 E) 2.8

Newton's Method for Finding Roots

50. If Newton's method is used to approximate the real root of $x^3+x-1=0$, then a first approximation $x_1=1$ would lead to a <u>third</u> approximation of $x_3 =$ (Calculator)

A) 0.682 B) 0.686 C) 0.694 D) 0.750 E) 1.637

Related Rates Problems

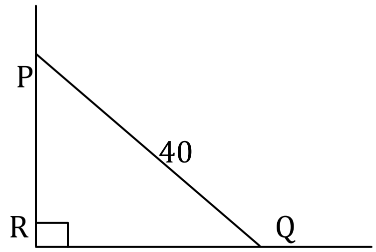


51. The sides of the rectangle above increase in such a way that $\frac{dz}{dt}=1$, and $\frac{dx}{dt}=3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

A) $\frac{1}{3}$ B) 1 C) 2 D) $\sqrt{5}$ E) 5

52. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

A) $-\frac{7}{8}$ feet per minute
B) $-\frac{7}{24}$ feet per minute
C) $\frac{7}{24}$ feet per minute
D) $\frac{7}{8}$ feet per minute
E) $\frac{21}{25}$ feet per minute



53. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

A) $\frac{6}{5}\sqrt{10}$ B) $\frac{8}{5}\sqrt{10}$ C) $\frac{80}{\sqrt{7}}$ D) 24 E) 32

54. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

A) $\frac{4}{27}$ B) $\frac{4}{9}$ C) $\frac{3}{4}$ D) $\frac{4}{3}$ E) $\frac{16}{9}$

55. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle? (Calculator)

A) A is always increasing.
B) A is always decreasing.
C) A is decreasing only when $b < h$.
D) A is decreasing only when $b > h$.
E) A remains constant.

56. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

A) $\frac{1}{\pi}$ B) $\frac{1}{2}$ C) $\frac{2}{\pi}$ D) 1 E) 2

57. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

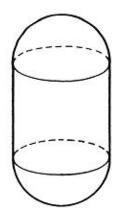
A) $-(0.2)\pi C$ B) $-(0.1)C$ C) $-\dfrac{(0.1)C}{2\pi}$ D) $(0.1)^2 C$ E) $(0.1)^2 \pi C$

58. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

A) 6 B) 8 C) 16 D) $4\sqrt{3}$ E) $12\sqrt{3}$

59. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

A) $\frac{1}{2}\pi$ B) 10π C) 24π D) 54π E) 108π



60. (1985 AB5) The balloon shown above is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius r and height h is $\pi r^2 h$, and the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.)

a) At this instant, what is the height of the cylinder?
b) At this instant, how fast is the height of the cylinder increasing?

61. (1988 BC3) An observer 100 meters away from the liftoff point is watching a balloon rise at a constant rate of 3 meters per second.

a) Find the rate of change in the distance between the observer and the balloon at the instant when balloon is 50 meters off the ground.
b) Find the rate of change in the area of the right triangle when the balloon is at 50 meters.
c) Find the rate of change of the angle of inclination of the balloon at 50 meters.

AP Calculus BC
Chapter 4 – AP Exam Problems

Answers

1.	A	1988	AB	#33	48%
2.	D	1998	AB	#24	52%
3.	E	1998	BC	#23	31%
5.	C	1993	AB	#23	21%
6.	B	1998	AB	#80	48%
7.	C	1993	AB	#27	65%
8.	C	1998	AB	#22	69%
9.	B	1993	BC	#22	75%
10.	C	1998	BC	#1	80%
11.	B	1985	AB	#16	73%
12.	A	1985	BC	#2	65%
13.	B	1993	BC	#14	75%
14.	C	1993	AB	#44	18%
15.	B	1998	AB	#89	38%
16.	B	1988	BC	#19	48%
17.	E	1988	AB	#4	60%
18.	C	1993	AB	#21	40%
19.	D	1998	AB	#1	78%
20.	C	1998	AB	#19	28%
21.	B	1988	BC	#27	39%
22.	C	1998	BC	#16	37%
23.	D	1993	BC	#9	70%
24.	C	1988	AB	#20	41%
30.	B	1988	AB	#8	59%
31.	D	1998	AB	#17	33%
32.	A	1998	AB	#76	61%

33.	A	1998	AB	#79	47%
34.	C	1985	BC	#20	79%
35.	E	1985	BC	#43	76%
37.	D	1993	AB	#18	40%
38.	B	1985	BC	#13	51%
39.	D	1988	BC	#24	55%
41.	B	1985	AB	#33	55%
42.	A	1998	AB	#23	68%
43.	D	1988	BC	#9	64%
44.	E	1998	BC	#6	83%
45.	C	1998	BC	#88	75%
46.	D	1988	AB	#45	43%
47.	B	1988	BC	#45	44%
48.	E	1993	BC	#36	44%
49.	D	1998	BC	#92	65%
50.	B	1993	AB	#45	47%
51.	B	1988	AB	#40	50%
52.	D	1993	AB	#34	28%
53.	E	1993	BC	#34	18%
54.	D	1988	BC	#37	43%
55.	D	1998	AB	#90	36%
56.	D	1993	AB	#39	42%
57.	B	1998	AB	#78	46%
58.	A	1985	BC	#22	69%
59.	C	1985	AB	#31	52%